

Let's Investigate
Lesson
2-3

Understand Proportional Relationships

KEY CONCEPTS

- Mathematics Objective**
- Determine whether quantities are proportional by testing for equivalent ratios.
- Language Objective** Describe how to test for equivalent ratios and explain, orally and in writing, whether quantities are in a proportional relationship.
- Essential Understanding** Quantities in a proportional relationship can be described by equivalent ratios.

COHERENCE

- In Grade 6, students:**
- found equivalent ratios.
 - used ratio and rate reasoning to solve real-world problems.
- In this lesson, students:**
- look for equivalent ratios to identify proportional relationships.
 - use proportions to solve problems.
- Later in this topic, students will:**
- use the constant of proportionality to write equations for proportional relationships and graph a line through the origin.
 - decide whether they can use proportional reasoning to solve given problems.

BALANCE

- Conceptual Understanding** Students extend their prior understanding of a unit rate and explore its relationship to equivalent ratios.
- Application** Students look for equivalent ratios to determine if a relationship is proportional.

New Vocabulary

- proportional relationship
- proportion

Materials

- centimeter grid paper
- centimeter ruler

Student Resources

- Family Engagement

Intervention System (IS)
M35 Solving Proportions

Teacher Resources

- Editable Lesson Plan
- Lesson Presentation Slides
- Slow Reveal Slides
- Sample Student Work
- Exit Ticket and Teacher Support
- Using Manipulatives: Understand Proportional Relationships: Equivalent Ratios
- enVision on the Go: Planning Support



Texas Essential Knowledge and Skills (TEKS)

The student applies mathematical process standards to represent and solve problems involving proportional relationships. The student is expected to:

7.4D* Solve problems involving ratios, rates, and percents, including multi-step problems involving percent increase and percent decrease, and financial literacy problems.

Also 7.7A*

*Readiness Standard

Mathematical Process Standards

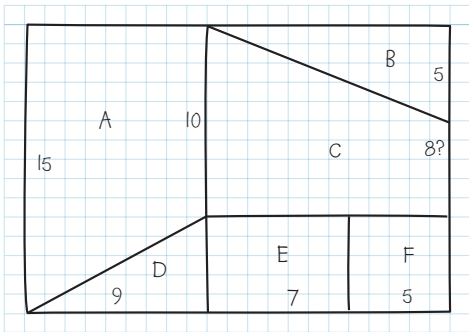
- The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
- 7.1D** Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.
- 7.1G** Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
- Targeted ELPS Support WRITING 4E**
- College and Career Readiness Standards VII.B.1, III.D.3, II.D.2**

Step 1 Investigate *continued*

Sample Student Work Available on Savvas Realize®

Student 1 Work

- incorrectly adds 3 to each measurement

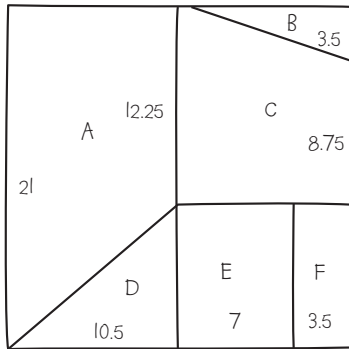


The difference from 4 to 7 is 3, so we added 3 to each length and then tried to draw it, but one length should be 8 long and there were only 5 squares left for it.

Ask: Adding 3 to 4 is almost doubling the measurement. Is adding 3 to 12 also almost doubling the measurement? [The pieces no longer fit together because they are increasing each by the same quantity, not percent.]

Student 2 Work

- correctly multiplies each length by 1.75



We found that to get 7 from 4, we had to multiply 4 by 1.75, so we multiplied every measurement by 1.75 and it worked.

Ask: Why did you decide to multiply 4 by a number to get 7? [All measurements needed to enlarge the same number of times.]

Student 3 Work

- incorrectly relates two ratios

$$\begin{array}{ccccc} \frac{4}{7} = \frac{x}{2} & \frac{4}{7} = \frac{x}{5} & \frac{4}{7} = \frac{x}{6} & \frac{4}{7} = \frac{x}{7} & \frac{4}{7} = \frac{x}{12} \\ 7x = 8 & 7x = 20 & 7x = 24 & 7x = 28 & 7x = 48 \\ x \approx 1.14 & x \approx 2.86 & x \approx 3.43 & x = 4 & x \approx 6.86 \end{array}$$

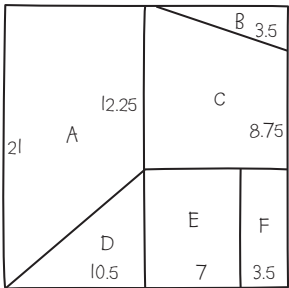
We used a proportion to convert each measurement but it didnt work because the lengths got smaller.

Ask: Do these ratios make sense? [The ratio of 4 to 7 relates the original to the new measurement, so the other ratios should do the same.]

Student 4 Work

- correctly relates two ratios

$$\begin{array}{ccccc} \frac{4}{7} = \frac{2}{x} & \frac{4}{7} = \frac{5}{x} & \frac{4}{7} = \frac{6}{x} & \frac{4}{7} = \frac{7}{x} & \frac{4}{7} = \frac{12}{x} \\ 4x = 14 & 4x = 35 & 4x = 42 & 4x = 49 & 4x = 84 \\ x = 3.5 & x = 8.75 & x = 10.5 & x = 12.25 & x = 21 \end{array}$$



We used the ratio of 4 to 7 to figure out the new lengths and then drew it.

Ask: How did you determine where to put each number in the ratios? [Relate the measurements in the same order—smaller over larger equals smaller over larger, or vice versa.]

Student 5 Work

- correctly writes an equation

$$\text{new length} = \frac{7}{4} \cdot \text{old length}$$

$$n = \frac{7}{4}x$$

$$n = \frac{7}{4} \cdot 3 = \frac{21}{4} = 5\frac{1}{4}$$

$$\frac{7}{4} \cdot 8 = 7 \cdot 2 = 14$$

$$\frac{7}{4} \cdot 1 = \frac{7}{4} = 1\frac{3}{4}$$

Ask: Is using an equation better than using a proportion? [The equation still uses a proportion, with the unknown isolated.]

Step 1 Investigate *continued*

Monitor SMALL GROUP

Monitor Students at Work Ask questions to assess and advance students’ thinking.

Assessing Questions

Questions for Students to Understand Their Thinking

- What operations might you use to enlarge measurements?
- What does the drawing tell you?
- How can you use the relationship between 4 units and 7 cm to enlarge the other measurements?
- Why do you think you can’t complete your puzzle?
- What could you try instead?

Questions for You as You Observe

- Which groups are struggling to relate the given measurements to a new pair of measurements?
- Which groups understand that they can increase each number in a similar way by using multiplication or a ratio?

Advancing Questions

Questions to Ask Students Who Need Help

- What is another operation that you could try?
- How many times greater than 4 is 7?
- How is this like zooming in on/enlarging a picture?

Questions to Help Students Think More Deeply

- How is a scale on a map like what you have done to enlarge the puzzle?

Select and Sequence Solutions

SMALL GROUP

Select and Sequence Students’ Solutions for presentations with the goal of building understanding of key math concepts.

- **Start** with a correct solution that multiplies each measurement by the same number to create the enlargement (Student 2 Work).
- **Then** show solutions that show incorrect strategies (Student 1 and Student 3 Work) and address any misconceptions.
- **Finally**, show other correct solution strategies such as using ratios and equations (Student 4 and Student 5 Work) and connect those representations to proportional reasoning.

Discuss Student Solutions WHOLE CLASS

After allowing students to finish the questions for the lesson, return to an all-class discussion.

- What is an important step in enlarging the puzzle so that the pieces still fit together correctly? [Students should understand that each larger measurement has been increased the same number of times or by using the same ratio.]
- How do you find the missing measurements once you know the ratio? [Students might answer by writing equivalent ratios or by using the ratio as a multiplier for each measurement.]
- How can you recognize a proportional relationship? [Students might answer that the two ratios are equivalent, or they see how the numerator and denominator in one fraction can be easily multiplied by the same number to create the other fraction.]

Targeted ELPS Support WRITING 4E

Pre-Production Review the Connect Mathematical Ideas direction. Read pairs of ratios to students such as $\frac{2}{4}$ and $\frac{3}{6}$. Give a thumbs up if I read equivalent ratios.

Beginning Review the Connect Mathematical Ideas direction. Ask students to name equivalent ratios and ratios that are not. Write *Equivalent* and *Not Equivalent*.

Intermediate Review the Connect Mathematical Ideas direction. Complete these sentences: An equation with equal ratios is a _____. [proportion] Equivalent ratios are _____. [proportional]

High Intermediate Review the Connect Mathematical Ideas direction. Ask students what is an equation with equal ratios and what relationships do quantities have when the ratios are equivalent.

Advanced Review the Connect Mathematical Ideas direction. Have students define ratio, and explain how ratios can be used to identify whether quantities are proportional.

Step 2 Connect



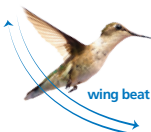
25-30 min

Choose a Problem

A How is the number of game lives awarded related to the number of gems found?



B How many times will the hummingbird beat its wings in 60 seconds?



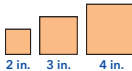
Hummingbird Wing Beats

Seconds (x)	2	7	10
Wing Beats (y)	160	560	800

C Dairy farmers produce milk to support the local economy. Is the ratio of dairy farmers to the number of cows milked proportional? Explain.

x	2	3	5	8
y	5	7.5	12.5	18

D Is the relationship between the area and the side length of the squares proportional? Explain.



Sample Student Work

A. Sample answer: Each ratio $\frac{\text{gems}}{\text{game lives}}$ is equivalent to $\frac{1}{3}$. The number of game lives awarded is proportional to the number of gems found.

B. Sample answer: Verify that the quantities are proportional.
 $\frac{160}{2} = \frac{80}{1}$ $\frac{560}{7} = \frac{80}{1}$ $\frac{800}{10} = \frac{80}{1}$
The ratios are equivalent, so the quantities are proportional.

Write and solve a proportion.

$$\frac{80}{1} = \frac{y}{60}$$
$$\frac{80}{1} \cdot 60 = \frac{y}{60} \cdot 60$$
$$4,800 = y$$

A hummingbird beats its wings 4,800 times in 60 seconds.

C. Sample answer: $\frac{5}{2} = \frac{7.5}{3} = \frac{12.5}{5} = 2.5$, but $\frac{18}{8} = 2.25$, so the ratio of dairy farmers and cows milked is not proportional.

D. Sample answer: The ratios are not equivalent, so the relationship between the area and side length is NOT proportional.

Side Length (x)	Area (y)	$\frac{\text{Area (y)}}{\text{Side Length (x)}}$
2	4	$\frac{4}{2} = \frac{2}{1}$
3	9	$\frac{9}{3} = \frac{3}{1}$
4	16	$\frac{16}{4} = \frac{4}{1}$

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Lesson 2-3 63

Choose a Problem INDIVIDUAL SMALL GROUP

Discuss Solution Strategies

Group students based on the chosen problem to discuss their solution strategies. Consider having groups record their solutions on a vertical surface. Do a gallery walk so they can discuss different problems and solution strategies.

Problem A

- What ratio can you write to compare the number of gems to the number of game lives?

Problem D

Is it necessary to find the area of every square?

Present and Analyze

Prioritize discussion about ratios and proportional relationships. If needed to promote discussion, ask questions such as:

Problem A

- Is the ratio of gems to game lives the same in every column? [Yes. In every column there are 3 game lives for every gem.]

Problem B

- How can you tell how many times the hummingbird will beat its wings in 1 second? [Simplify the ratio in the first column of the table: $\frac{2}{160} = \frac{1}{80}$. The hummingbird beats its wings 80 times in 1 second.]

Problem C

- How can you tell if the relationship is not proportional? [If the relationship is not proportional, the ratio of dairy farmers to cows milked will not be the same in every example.]

Problem D

- How can you use ratios to test if the relationship is proportional? [First find the area of each square and then set up ratios using each side length and area to test if they are equal.]

Engage Students Through Choice

Assist students in selecting a problem to solve individually.

Problem A is for students who are still working on understanding the result of multiplying or dividing by a constant value.

Problem B is for students who need more work on recognizing sets of equivalent ratios.

Problem C is for students who are ready to apply understanding of equivalent ratios in a proportional relationship.

Problem D is for students who understand proportionality and will benefit from using their reasoning to identify a nonproportional relationship.

Early finishers may choose more than one problem.

Step 2 Connect *continued*

Connect

1. **Communicate Reasoning** How was the strategy you used in *Choose a Problem* the same as your strategy in *An Even Bigger Puzzle*? How were your strategies different?

You compare ratios in each problem.

A. The ratios are all equivalent.

B. Since all of the ratios are equivalent, you can apply proportional reasoning.

C. Some ratios were equivalent, but one was not, so the relationship is not proportional.

D. Although there was a pattern, the ratios were not equivalent so there is not a proportional relationship.

2. **Analyze Math Relationships** How do you know if a relationship between two quantities is NOT proportional?

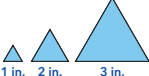
The ratios $\frac{y}{x}$ for all pairs of x and y will not be equivalent.

3. **Evaluate Reasonableness** If the ratio $\frac{y}{x}$ is the same for all related pairs of x and y , what does that mean about the relationship between x and y ?

It means that x and y have a proportional relationship.

4. **Analyze Math Relationships** Each triangle is equilateral. Is the relationship between the perimeter and the side length of all equilateral triangles proportional? Explain.

Yes; the quantities are proportional because the ratios $\frac{3}{1}$, $\frac{6}{2}$, and $\frac{9}{3}$ are equivalent.



Key Concept

If a set of numbers is multiplied or divided by a constant factor, the comparison of the original set of numbers to the resulting set of numbers is a set of equivalent ratios.

For example, multiplying each element of {1, 3, 5} by 2 gives {4, 12, 20} and $\frac{1}{4} = \frac{3}{12} = \frac{5}{20}$. A set of equivalent ratios represents a proportional relationship.

Talk About Math Ideas

How does a picture on a phone camera show proportionality? What is another example of a proportional relationship in your world?

64 Lesson 2-3

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Connect  INDIVIDUAL  WHOLE CLASS

Connect Solution Strategies Use your students’ work or the sample student work to facilitate responses phrased in students’ own words. Encourage students to use the vocabulary *equivalent ratios*, *proportion*, and *proportional relationship*.

Item 1 Comparing and contrasting different solutions builds understanding of problems that involve proportional relationships. Have students focus on their use of equivalent ratios.

Item 2 Compare equivalent ratios to unit rates. The unit rate defines the relationship between quantities. All ratios that relate the quantities in a proportional relationship can be simplified to the unit rate.

Item 3 Introduce the idea of the ratio $\frac{y}{x}$ as a constant. If two quantities do not have equivalent ratios, they will not be proportional.

Item 4 Discuss why this is true of all equilateral triangles, not just the three shown in the problem. *What about an equilateral triangle with a side length of 10 inches? x inches? Can you write an equation that models this relationship?*

Key Concept

- How can you use structure to help you decide whether two quantities have a proportional relationship? [If you calculate the ratio $\frac{y}{x}$ for each data pair and they are equivalent, then the two quantities have a proportional relationship.]
- When testing ratios to see if they are equivalent, why is order important? [Ratios must be created by pairs of numbers that are related to each other in the situation (e.g., corresponding sides in a shape) and those numbers must be used in the same order in the ratios.]

Talk About Math Ideas

Have students engage in a think-pair-share to discuss the response to the question about phone cameras and proportionality. Listen for discussions about proportionality maintaining the same multiplicative relationship, which maintains the shape of an object that has been enlarged or reduced, or relative values in two sets of data.

Lesson 2-3 64

EP

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Step 3 Practice and Problem-Solving



15-20 min



Interactive Practice



Adaptive Practice

Name _____

Practice and Problem Solving



5. The amount of seed a landscaper uses and the area of lawn covered have a proportional relationship. Complete the table.

Lawn Seed

Seed (oz)	2	3	4
Area Covered (ft ²)	50	75	100
Area Covered Seed	$\frac{50}{2} = \frac{25}{1}$	$\frac{75}{3} = \frac{25}{1}$	$\frac{100}{4} = \frac{25}{1}$

6. **Communicate Math Ideas** Is the relationship between the number of slices of salami in a sandwich and the number of Calories in the sandwich proportional? Explain.

Calories in a Sandwich

Slices of Salami	Calories
1	66
2	96
3	126
4	156

No; the ratios of Calories to slices of salami are not all equivalent.

7. **Connect Math Ideas** A wholesale club sells eggs by the dozen. Does the table show a proportional relationship between the number of dozens of eggs and the cost? Explain.

Cost of Dozens of Eggs

Dozen	Cost (\$)
6	21
8	28
10	35
14	49

Yes; the ratios of cost to dozens of eggs for all related pairs are equivalent.

8. Does the table show a proportional relationship? If so, what is the value of y when x is 11?

x	4	5	6	10
y	64	125	216	1,000

No; the quantities x and y do not have a proportional relationship, so you cannot use equivalent ratios to find the value of y when x is 11.

9. Does the table show a proportional relationship? If so, what is the value of y when x is 10?

x	5	6	7	8
y	$1\frac{2}{3}$	2	$2\frac{1}{3}$	$2\frac{2}{3}$

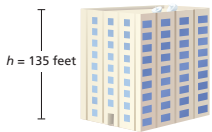
Yes; $y = 3\frac{1}{3}$

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Lesson 2-3 65

You may opt to have students complete automatically scored Practice and Problem Solving or Mixed Review online.

10. The height of a building is proportional to the number of floors. The figure shows the height of a building with 9 floors.



- a. **Evaluate Reasonableness** Write the ratio of height of the building to the number of floors. Then find the unit rate, and explain what it means in this situation.

$\frac{135 \text{ ft}}{9 \text{ floors}} = \frac{15 \text{ ft}}{1 \text{ floor}}$. Each floor of the building is 15 ft high.

- b. How tall would the building be if it had 15 floors?
225 ft

11. **Higher Order Thinking** Do the two tables show the same proportional relationship between x and y? Explain how you know.

Yes; All the ratios of y to x are equivalent. Every ratio $\frac{y}{x}$ is equivalent to $\frac{3}{4}$.

x	160	500	1,200
y	360	1,125	2,700

x	2	5	7
y	4.5	11.25	15.75

STAAR Assessment Practice

12. The table shows the number of cell phone towers a company will build as the number of its customers increases. 7.4D

PART A

Is the relationship between the number of towers and the number of customers proportional? Explain.

Yes; The ratios of towers to customers are all equivalent.

PART B

If there are 576 towers, how many customers does the company have? Create a proportion and solve.

12,000 customers

Cell Phone Towers

Customers (thousands)	Towers
5.25	252
6.25	300
7.25	348
9.25	444

13. Select all true statements about the table. 7.4D

- ☒ The table shows a proportional relationship.
☐ When x is 20, y is 2.55.
☐ All the ratios $\frac{y}{x}$ for related pairs of x and y are equivalent to 8.
☒ The unit rate of $\frac{y}{x}$ for related pairs of x and y is $\frac{1}{8}$.
☒ If the table continued, when x is 30, y would be 3.75.

x	12	18	22	26
y	1.5	2.25	2.75	3.25

66 Lesson 2-3

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Practice and Problem Solving

Engage Through Student Choice

Go online for an assignment strategy.

Error Intervention

Item 6 Students may see a pattern that shows that each slice of salami adds 30 Calories. Remind them that patterns can exist, but for the relationship to be proportional, the ratios of Calories to slices of salami have to be equivalent.

- What is the ratio of the number of Calories to the number of slices of salami in a sandwich with 1 slice? 2 slices? Are the ratios equivalent? [$\frac{66}{1}$, $\frac{96}{2} = \frac{48}{1}$; No]

Emergent Bilingual Student Support

Item 7

- What is unit rate in this problem? [The cost of 1 dozen eggs, or \$3.50]

65-66 Lesson 2-3

Item Analysis: Practice and Problem Solving

Item	DOK	TEKS	
		Content	Process
5, 8	1	7.4D	
6	1	7.4D	7.1D
7	1	7.4D	7.1F
10	1	7.4D	7.1B
9, 11-13	2	7.4D	

Quick Check



- Assess lesson content with the Quick Check.
- The online Quick Check is **automatically scored** and **remediation, practice, or enrichment** will be assigned based on student performance.
 - You can use the Assessment Sourcebook version to prescribe targeted remediation or enrichment.

Item	DOK	TEKS	
		Content	Process
1, 2, 4	1	7.4D*	7.1B, 7.1E
3, 5	2	7.4D*	7.1A, 7.1F

0–3 points: Reteach to Build Understanding and Additional Vocabulary Support Activities

4–5 points: Enrichment Activity

In addition, **emergent bilingual students** may benefit from the Build Mathematical Literacy Activity.

Differentiation Library

- Additional Differentiation Library resources are described at the start of this topic.
- Differentiation resources that support diverse learning styles are online and in the Differentiation Library Teacher’s Guide.

Video Tutorials Available on Savvas Realize®

Students can access these tutorials anytime to reinforce lesson content.

-  Determine Whether Values in a Table are Proportional
-  How Do You Know if Two Ratios are Proportional?

Item Analysis: Additional Practice

Item	DOK	TEKS	
		Content	Process
1	1	7.4D	
2, 3	1	7.4D	7.1G
4–9	2	7.4D	

Additional Practice

Printable or autoscored on Savvas Realize®

Name _____

2-3 Additional Practice

1. At a café, the cook uses a recipe that calls for eggs and milk. The amounts of eggs and milk have a proportional relationship. Complete the table.

Ingredients in Recipe			
Number of Eggs	2	3	4
Cups of Milk	6	9	12
Milk Eggs	$\frac{3}{1}$	$\frac{3}{1}$	$\frac{3}{1}$

2. Explain Math Ideas

Is the relationship between x and y proportional? Explain.

x	y
5	25
6	30
7	35
8	40

Yes; Sample answer: All the ratios of y to x are equivalent. Every ratio $\frac{y}{x}$ is equivalent to 5.

3. Justify Math Arguments

Does the table show a proportional relationship between x and y ? Explain.

x	y
2	4
4	16
7	79
10	100

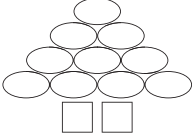
No; Sample answer: All the ratios $\frac{y}{x}$ are not equivalent.

4. Does the table show a proportional relationship? If so, what is the value of y when x is $\frac{3}{5}$?

x	y
30	150
$\frac{1}{6}$	$\frac{5}{6}$
199	995
$\frac{2}{15}$	$\frac{2}{3}$

Yes; $y = 3$

5. In a stationery design, the number of ovals is proportional to the number of squares. How many squares will there be when there are 75 ovals?



15 squares

15 2-3 Let's Investigate! Understand Proportional Relationships

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6. The table shows a proportional relationship between x and y .

a. Complete the table.

x	y	$\frac{y}{x}$
3	30	$\frac{30}{3} = \frac{10}{1}$
5	50	$\frac{50}{5} = \frac{10}{1}$
7	70	$\frac{70}{7} = \frac{10}{1}$

b. Katerina says the ratio $\frac{y}{x}$ is $\frac{1}{10}$. What error did she likely make?
She found $\frac{y}{x}$ instead of $\frac{x}{y}$.

7. Higher Order Thinking

Do the two tables show the same proportional relationship between x and y ? Explain.

x	500	750	1,000
y	1,250	1,875	2,500

x	3	4	5
y	4.2	5.6	7

No; Sample answer: The first table shows $\frac{y}{x} = 2.5$. The second table shows $\frac{y}{x} = 1.4$. $2.5 \neq 1.4$.

STAAR Assessment Practice

8. During snowstorms, the city sends out trucks to plow. The amount of snowfall and the number of trucks sent out are shown in the table.

PART A

Is the relationship between the amount of snowfall and the number of trucks proportional? Explain.

Yes; the ratios of trucks to inches of snowfall are all equivalent to 2.5.

Snow Plowing Plan	
Snowfall (in.)	Trucks
6	15
8	20
12	30
18	45

PART B

For a 23-inch snowfall, how many trucks would the city send out? Explain.

Sample answer: For a 23-inch snowfall, the number of trucks would be 57.5. The city should send out 58 trucks to keep the roads safe.

9. Which of the following statements about the table is true? Select all true statements about the table.

☒ The table shows a proportional relationship.

☐ All the ratios $\frac{y}{x}$ for related pairs of x and y are equivalent to 7.5.

☒ When x is 13.5, y is 4.5.

☐ When y is 12, x is 4.

☒ The unit rate of $\frac{y}{x}$ for related pairs of x and y is $\frac{1}{3}$.

x	y
10.5	3.5
15.9	5.3
22.5	7.5
27	9

16 2-3 Let's Investigate! Understand Proportional Relationships

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